The Tree of Number

The Circlemath view of number is that number is organic. Like a living tree, one branch turns into another. Maths is not like a house with dead planks of wood nailed together, but intrinsically whole.

Multiplication	**Division**
Addition	**Subtraction**
Counting Up	**Counting Down**

The Tree of Number

Counting and Adding

We begin with simple counting. Counting splits into two, counting up and counting down. The rest of number inherits from this initial split, giving us two sides (the red and the yellow).

Counting always adds on (or subtracts) one at a time. It is 1, 2, 3... or ...3, 2, 1

The new innovation that addition has over counting is that addition can add in irregular jumps. It doesn't have to march forward one at a time.

Example of Counting up: 5, 6, 7, 8

Example of Addition: 5 + 3 = 8

If we said "69 + 3" you might COUNT out "70, 71, 72". That's counting.

But if we said "5 + 100" you would ADD them to get 105. You wouldn't count "6, 7, 8... " up to 105!

Counting up and Adding organically turn into one another.

Here is an addition:
5 + 3 = 8
Here is another:
5 + 2 = 7
Here is another:
5 + 1 = 6
Or is that counting?

Adding on 1 is the same as counting. That is where they blend and merge and are indistinguishable from one another. One turns organically into the other. Likewise, the mirror, counting down becomes subtraction.

Adding and Multiplication

Here are some addition sums:

5 + 3 + 2 = 10

4 + 1 + 7 = 12

2 + 2 + 2 = 6

The last one ***adds*** the ***same number*** to itself 3 times. We can shorthand it to "3 x 2 = 6" which means "three two's ADD to 6".

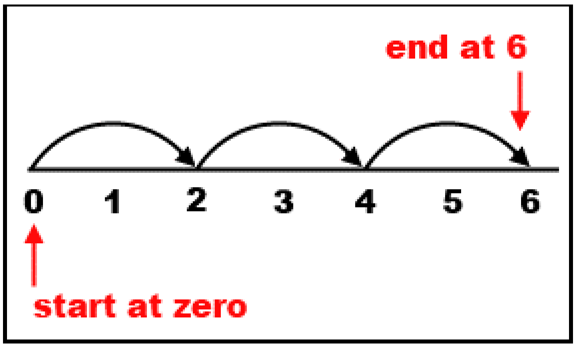
You can see that multiplying grows organically out of adding.

Multiplying is repeat addition. We add the same number repeatedly again and again a certain number of times. Hence the term "the times tables".

In multiplying we have returned to the uniformity found in counting, except instead of adding on only 1 at a time we can add on whatever number we like.

Turn these into multiplications:

5 + 5 + 5 + 5 = 20	becomes	_____	x	_____	= 20
8 + 8 + 8 + 8 + 8 = 40	becomes	_____	x	_____	= 40
7 + 7 + 7 = 21	becomes	_____	x	_____	= 21



3 x 2 = 6

How many jumps? There are 3 jumps.

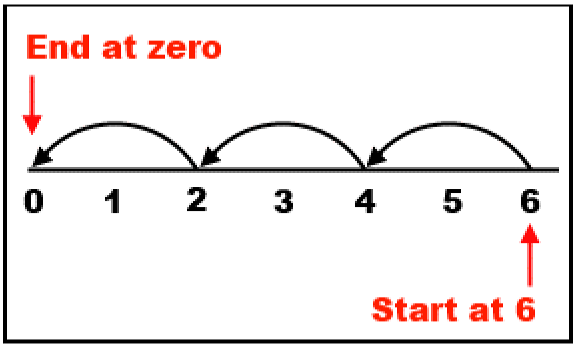
How big is each step? Each step is of size 2.

Note:

I prefer to say "3 x 2 = 6" means "three two's are six". There is no doubt then what you mean. If you say "three times two is six" then it could mean "take three" and "times it by two" to get six OR it could mean "three times" the "the number two" is six.

Division and Subtraction

In exactly the same way subtraction organically turns into division.



6 ÷ 2 = 3

Start from 6

The step size is 2. Take away 2 at a time.

End at zero.

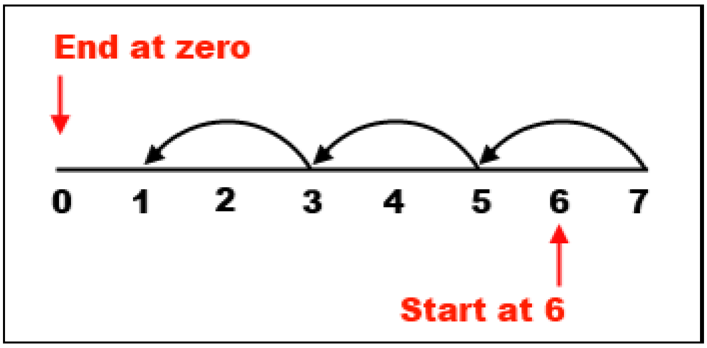
How many steps did you take? The answer is 3.

Compare this to the multiplication picture and you will see that it is exactly the same except running in reverse order!

The end point of 3x2 = 6 is the start point for the division 6 ÷ 2 = 3. The start point for 3x2 = 6 is the stop point for the division. The division answers "how many times can 2 be subtracted from 6 before reaching zero?"

Remainders

A division can have a remainder. For example 7 ÷ 2 = 3 r 1



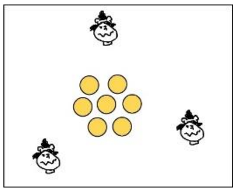
You can't take another 2 off after you reach 1 without going past zero so you must stop.

There is 1 remainder.

It is not yet divided, it is "left over"

Stone Age Division

Let's go back in time to an early tribal scene.



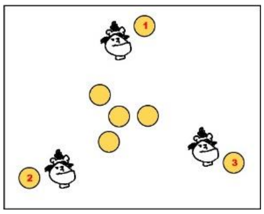
Seven corn pancakes have been rolled out.

They are to be equally divided by 3 stone-age people.

We will record the process pictorially, via "repeat subtraction" and by the usual long-division process so you can see how they compare:

Step 1:

They each take 1 pancake.

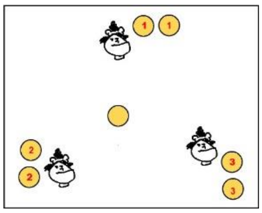


That's:

```
3 ) 7
    4        //Take 3 off 7 = 4
```

Step 2:

There's enough there for another pancake each:



That's:

```
3 ) 7

    4

    1        //Take another 3 off the 4 = 1
```

Step 3:

They can't share it out any further as you can't take 3 off 1. So they stop.

TWO threes came out of the 7 leaving a remainder of 1 "left over".

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   2                                 2
3) 7                 OR           3) 7
   4                              -  6
   1    <---remainder                1    <----remainder
```

In the standard algorithm (above right) we "guess" there are about two 3's in 7 so we work out what two 3's comes to (the 6) and take it away to find the remainder.

On the left we count the times we took 3 away (one for the 4, one for the 1, that's two) and the remainder is what we have when we can't take away any more.

Fractions

We will return to a simpler example: 7/2 = 3 r 1.

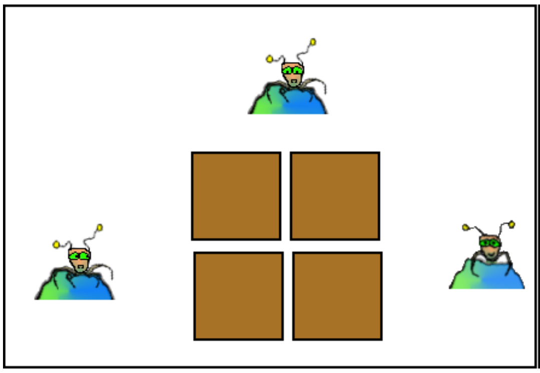
If we wanted to indicate that the remainder should also be divided by 2 then we could write the answer as 3 ½ instead of "3 r 1". The ½ means the remainder of 1 is to be divided by 2. And we could do this with a pair of scissors for example if we were talking about sheets of paper.

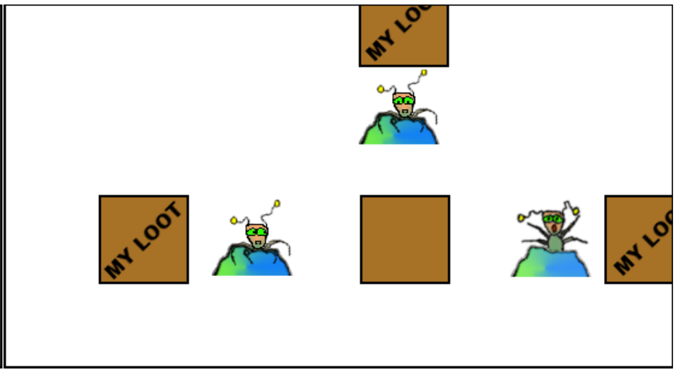
But if we were talking about 7 cute little bunny rabbits that were being divided up by 2 little girls, then the answer 3 ½ would ***not*** actually mean that we had taken a knife and cut the remaining bunny rabbit in half! So ½ can mean "the 1 remainder is yet to be divided by 2" in some manner.

So ½ can mean we have actually cut something in half or it can mean "the 1 is yet to be divided by 2"

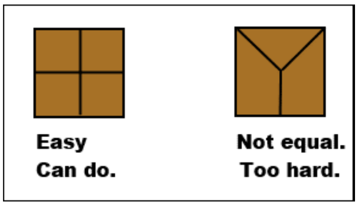
Decimals

Imagine 3 ants have found four square biscuits. They want to divide it up amongst themselves equally. Their first step is to each take a biscuit but that leaves 1 over that has not been divided:

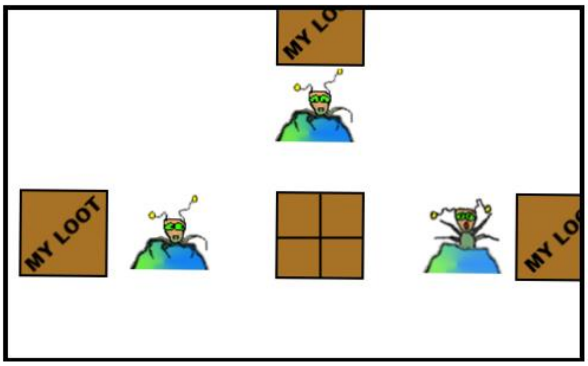


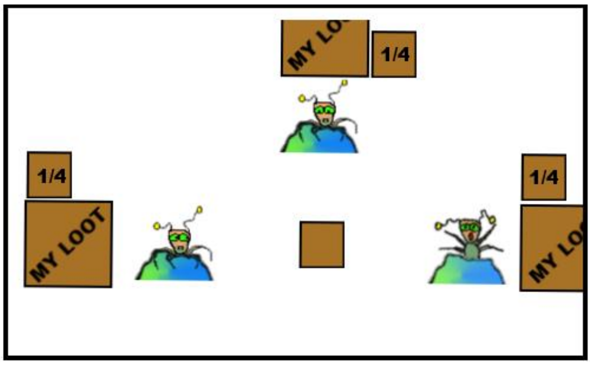


The answer is that they get 1 $^{1}/_{3}$ biscuits each. But they only have a knife and only know how to cut things into quarters. They can't actually cut it into exact thirds. Hey, they are just ants! They can cut into quarters easily enough. They can't figure out how to exactly cut it into thirds...

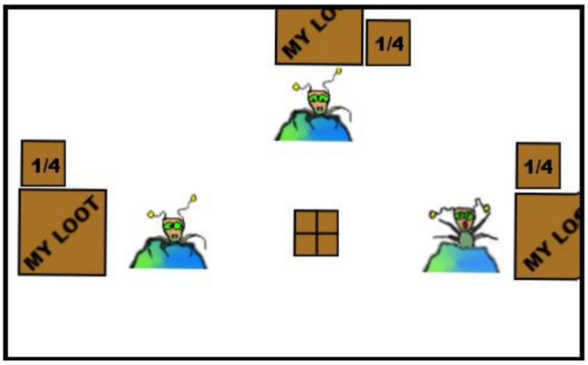


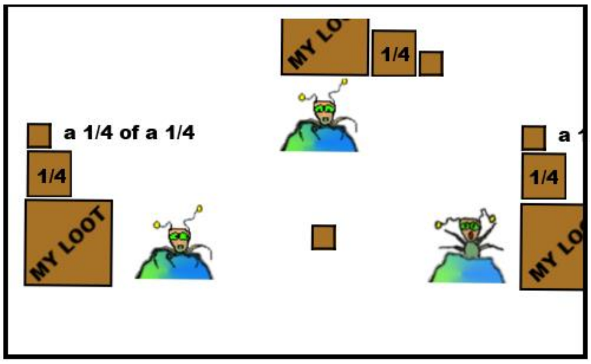
So they cut the remaining biscuit into quarters and divide it out again:



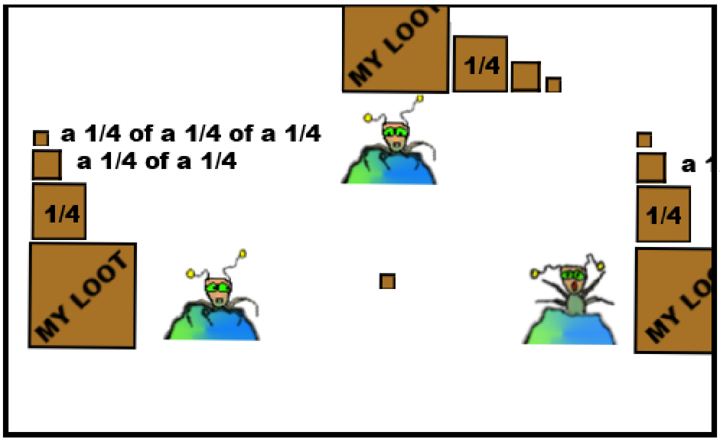


But darn it all. There is still a bit left over! And being greedy little ants they want to share that out equally too! Waste not want not! So they cut it into four pieces (as that's all they can do):





How far could this go on? Knowing ants, a very long time. Until the knife they are using to cut the crumb is wider than the crumb itself! Then I guess they just have to give up and throw the rest away.



Notice that they got 1 whole biscuit

and a 1/4 of a biscuit

and a 1/4 of a 1/4 of a biscuit

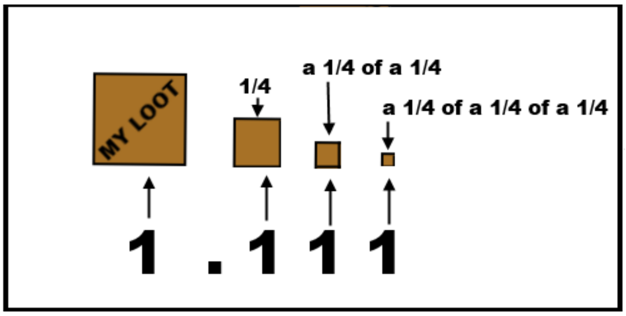
and if they did it again they would get

a 1/4 of a 1/4 of a 1/4 of a biscuit...
and so on.

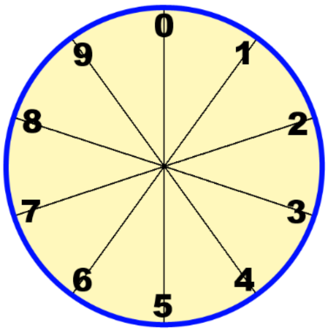
They could write it out this way:

They each get 1.111... biscuits.

Where the number after the decimal point in their language ("Antlish")means a fraction of a biscuit (1/4) or (1/4 of a 1/4) or (1/4 of a 1/4 of a 1/4):



In maths today we do virtually the same. Our "cookie-cutter" doesn't cut things into fourths however, it cuts things into tenths!



Our cookie cutter

cuts into 10 equal parts

at a time.

If we had to divide 10 round corn pancakes by 9 hungry people, we would first of all give everyone 1 pancake each. That would take care of 9 whole pancakes. But there one left over. A remainder...

Like the ants, we apply our cookie cutter to it and divide the remaining pancake into ten equal parts (tenths).

Like the ants we give each person one of those tenths. Everyone now has 1 and $^{1}/_{10}$ of a pancake. That used up $^{9}/_{10}$ of the pancake.

But like the ants we are left over with a tiny bit remaining. 1 whole tenth is left...

Like the ants we take that bit left over and cut it up into ten equal parts our cookie cutter.

We cut the $^{1}/_{10}$ into tenths. So now each bit is $^{1}/_{10}$ of a $^{1}/_{10}$. That's $^{1}/_{100}$ ths.

Repeat the process, give everyone 1 of these "$^{1}/_{10}$ ths of $^{1}/_{10}$ths" and everyone ends up with $^{1}/_{100}$ th of a cookie as well.

Now everyone has 1 whole cookies, 1 tenths of a cookie and 1 hundredth of a cookie.

But there is still 1 part left over!

And on we go...

No - I don't think so! Let's throw the crumb away!

But note: The crumb still exists! If we were being strictly scientific instead of dealing with market place practicalities, we would acknowledge that we never can quite do that division!

We say each person got 1.111... cookies: