

Complement of Difference Strategy

You may have noticed that all the examples to date focus on repeat subtracting 9, 8 or 7. What about the other numbers? How about subtracting 6, 5, 4, 3 or 2?

The reason for this is that the "Add a Complement" strategy works best for 9's, 8's and 7's.

To show this, consider $32 - 3 = 29$. Using the "Add a Complement" strategy we:

The image shows three boxes illustrating the 'Add a Complement' strategy for the subtraction $32 - 3 = 29$.
Box 1: A vertical subtraction problem $32 - 3 = 2$. The result '2' is in red.
Box 2: A vertical subtraction problem $32 - 3 = 2$. A red arrow points from the '2' to a red '7' written to the right of the '2'.
Box 3: A vertical subtraction problem $32 - 3 = 29$. The result '29' is in red.

*Drop the "thirty"
down to "twenty"*

*Add the
complement of 3
(which is 7)
to the 2...*

...to get 9.

Ok...

It works.

But it's awkward.

I think I'd prefer to count backwards!

Fortunately

there is a

better way!

Introducing...

The Complement of the Difference Strategy

For those who just want to **use** the strategy without any complicated explanations, here's how to do it. The explanation for those wanting the reason behind the method follows after this page.

Take the Complement of the Difference
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Example 1:

$$\begin{array}{r} 85 \\ - 6 \\ \hline 79 \end{array}$$

"Comp of Diff" Strategy:

Step 1: Note the end digits of the sum, "5 - 6" go below zero. It is "hard".

Step 2: Drop the 8 of "eighty" down to 7 for "seventy". (A carry process will occur).

Step 3: Find the difference between 5 and 6. It is 1.

Step 4: Take the complement in 10-circle of 1. It is 9. That is the last digit.

*It will work with all examples,
but works best with numbers that are close together.*

Example 2:

$$\begin{array}{r} 62 \\ - 4 \\ \hline 58 \end{array}$$

"Comp of Diff" Strategy:

Step 1: Note the end digits of the sum, "2 - 4" go below zero. It is "hard".

Step 2: So drop the 6 of "sixty" down to 5 for "fifty". (A carry process will occur).

Step 3: Find the difference between 2 and 4. It is 2.

Step 4: Take the complement of 2. It is 8. That is the last digit.

The Busy Office

Imagine a busy international office. There are very big and important people rushing about trying to arrange international meetings with diplomats from other countries, the stock market exchange is buzzing away in the background influencing their decisions. Everyone is changing their mind about how to get to the meetings and what they will say and do. It's a very busy office.

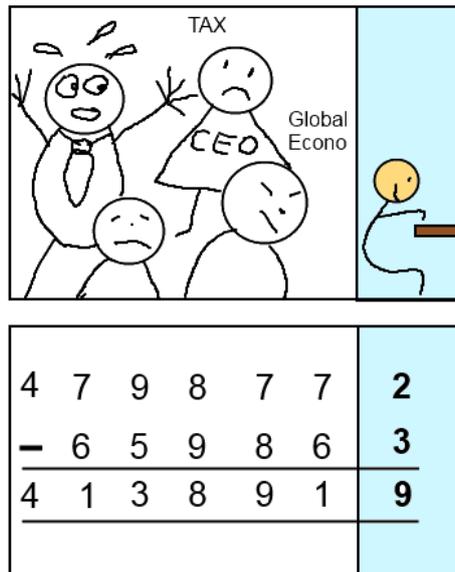
But out the back in a tiny office lives a lowly clerk. Her job is simply to buy the plane tickets for the various diplomats.

From her point of view she is completely shut off from the hubbub of the frenzied diplomats. All that happens for her is that a piece of paper arrives saying "we need two flights to Washington on the 25th". Ah... Peace at last. Her job is simple. She ignores everything else that is going on and simply makes the bookings that come to her. No further responsibility beyond that.

She is in a black box, isolated from the rest of the noise.

10'C is also a Black Box

The 10'c (short for 10-circle) is also a black box. It only cares about the last digit of any sum:



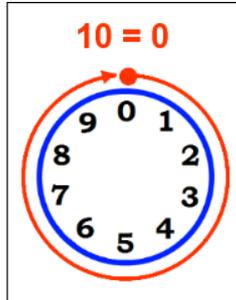
The sum can be as complicated as you like. But 10'c doesn't care. It just deals with the last digit. This is what it sees in its little world:

2
- 3
9

That's all it has to deal with.

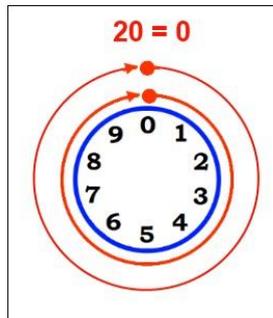
It can ignore all the rest of the noise from the big numbers because in its world it only has 10 little numbers it recognises, 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. That's it.

In 10's the number 10 is just thrown away. It becomes nought:



10's just doesn't care. It doesn't even see the 10.
It just sees the 0 at the end of the 10.
So it's 0.

Likewise if we count to 20 it throws away the 20 because that's just two 10s.
That's twice around 10's and back to 0.



You can see what's coming.

The count of 30 is 3 times around 10's and back to 0.

$$30 = 0$$

$$40 = 0$$

$$50 = 0 \quad 60 = 0 \quad 70 = 0 \quad 80 = 0$$

$$90 = 0$$

and so even

$$100 = 0$$

That means 200 = 0

$$300 = 0$$

$$400 = 0 \quad 500 = 0 \quad 600 = 0 \quad 700 = 0 \quad 800 = 0$$

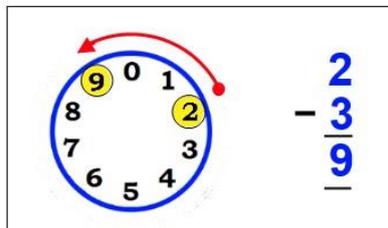
$$\text{even } 900 = 0$$

so

$1000 = 0$
 and so on and on...
 As far as 10^c is concerned
 the doors are closed.
 It works in its own tiny little world.
 $57,849,395,849,493,059,584,330$
 is just 0.

It only sees that last digit. That's all. That's all its concerned with.
 So if our sum is:

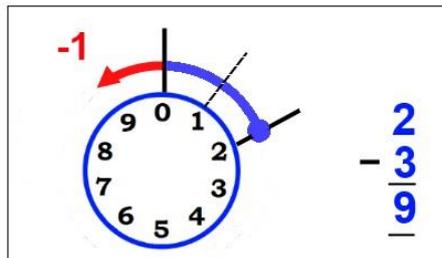
$$\begin{array}{r} 62 \\ - 3 \\ \hline 59 \end{array}$$



Then the little "clerk" in 10^c only sees $2 - 3 = 9$.
 Her only problem is to chart the best way to get from 2 to 9
 by going backwards 3 steps.
 That simplifies the task!
 Forget the rest!

In 10^c the sum "2 - 3" actually comes to -1
 But -1 is 9!

The answer we want!



It just so happens that

- 1 = 9
- 2 = 8
- 3 = 7
- etc.

And these are the complements!

Actual Practice

The method is to take
the complement of the difference.

In theory we say:

$2 - 3 = -1$ (that's the difference)
and $-1 = 9$ (that's taking the complement)

In practice we don't bother.

We see 2 and 3 are 1 apart.

We say "9"

And of course, we remember to bring the rest of the sum in.

So $62 - 3$ becomes 59

not just 9!

Frames of Reference

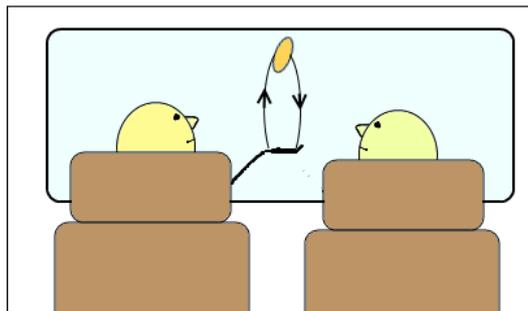
Here is a little puzzle for you.

You are in a car and your friend seated beside you tosses a coin in the air.

Heads or tails...

Who cares. It spins in the air for about a second and lands.

How far did it travel?



Let me see. The roof of the car is pretty low.

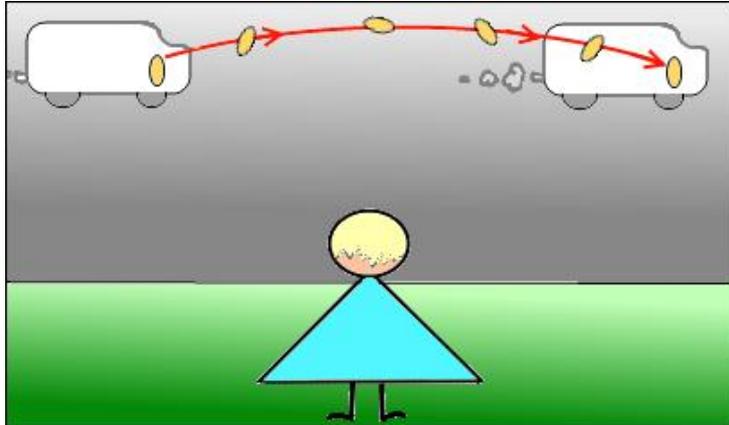
It can only go up about 50cm at max.

Then it falls back down another 50cm.

At the most the it travelled 100 cm right?

That's one meter total.

BUT
how far would it seem to travel
to someone standing on the grass at the side of the road
as the car whisks by at 100 kph?



In the same second
they would see the coin (through the window)
travel at least 100 meters!

So who is right and who is wrong?

If we say the people in the car were wrong, their measurement was an illusion,
then we must remember
that the Earth is travelling around the sun at about 30 km/s.
Yes. That's 30 kilometers for each second!
So that would mean the real distance the coin travelled was 30 km in that second!

And also don't forget the sun itself is travelling around the milky way
at an astonishing 230 kilometers per second!
So it's more like that coin travelled 230 km!

And our galaxy is moving relative to the rest of them...

So how far did it move?
1m, 100m, 30km, 230km or more?

Actually each of these answers is equally valid.
The answer is that it depends on the observer.
It depends on your
Frame of Reference

A circle is a **frame of reference** for numbers.
62 is just 2 from **the point of view of 10's**
In the frame of reference of 10's we see 62 as 2