

Lesson 1

Single Digit Subtraction

This is the quick guide to the video. For more complete details watch the video.

Goal:

To speed up and simplify single digit subtraction until answers can be produced as fast as one can write them down.

Method:

There are two classes of subtraction, one's which "go" and one's which "won't go". The one's that "won't go" cause all the trouble and are perceived as being "harder".

Examples:

$$\begin{array}{r} 56 \\ - 2 \\ \hline 54 \end{array}$$

$$\begin{array}{r} 48 \\ - 7 \\ \hline 41 \end{array}$$

$$\begin{array}{r} 56 \\ - 8 \\ \hline 48 \end{array}$$

$$\begin{array}{r} 48 \\ - 9 \\ \hline 39 \end{array}$$

Class A
"Easier"

class B
"Harder"

Looking at the LAST DIGITS only we see:

$$\begin{array}{r} 6 \\ - 2 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 8 \\ - 7 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 6 \\ - 8 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 8 \\ - 9 \\ \hline 9 \end{array}$$

Class A
"Easier"
Result is Above zero

class B
"Harder"
Result is Below zero

(or zero exactly)

Handling Class A Subtractions

The subtraction will go. The result will be **A**bove Zero (or zero exactly).
Just take away from the units digit:

Examples

$$\begin{array}{r} 56 \\ - 2 \\ \hline 54 \end{array} \qquad \begin{array}{r} 48 \\ - 7 \\ \hline 41 \end{array}$$

Units column: $6 - 2 = 4$ and $8 - 7 = 1$
Tens column: stays the same

This could be modelled with money.
For example $56 - 2 = 54$ would be:

5 ten dollar bills	and	6 one dollar coins
less		2 one dollar coins
5 ten dollar bills	and	4 one dollar coins

Of course, the trick is to know how to handle the Class B subtractions!
That's coming next.

Handling Class B Subtractions

The subtraction "won't go". That is, if it did go, it would go **B**elow Zero.

Examples

$$\begin{array}{r} 63 \\ - 9 \\ \hline 54 \end{array} \qquad \begin{array}{r} 47 \\ - 9 \\ \hline 38 \end{array}$$

Money Model:

$$\begin{array}{r} 63 \\ - 9 \\ \hline 54 \end{array}$$

6 ten dollar bills	and	3 one dollar coins
less		9 one dollar coins
is what?		

Step 1: Note you can't take 9 coins away from 3 coins.
You will need to break into a ten dollar bill

Step 2: Take away 1 ten dollar bill and give back 1 one dollar coin as change
That's the same as taking away 9

6 ten dollar bills	and	3 one dollar coins
<u>less</u>		<u>9 one dollar coins</u>
5 ten dollar bills	and	4 one dollar coins

Closer Focus on Method:

Step 1: Note you can't take 9 coins away from 3 coins.
You will need to break into a ten dollar bill

Step 2: Change 1 ten dollar bill for 10 one dollar coins
There are only 5 ten dollar bills left now.
We reduce the tens unit by 1:

6 ten dollar bills	and	3 one dollar coins
<u>less</u>		<u>9 one dollar coins</u>
5 ten dollar bills	and	what?

Step 3: Split the 10 one dollar coins into 9 you want to take away
and 1 you will leave behind

Step 4: Take away the 9 and add the 1 to the 3 to get 4 one dollar coins.

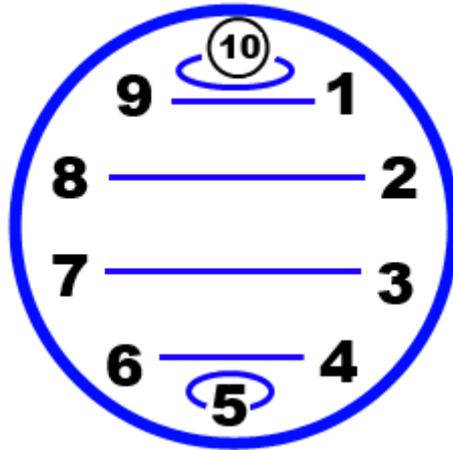
Add a Complement Method

Complements

Step 3 is always to change a \$10 bill into 10 one dollar bills, and then to split that into two components.

One is the part you are taking away and the other is what you will end up adding on to the rest of the units.

In Circlemaths we call these two numbers complements of one another.



These are the complements in a 10-circle

0 and 10
 9 and 1
 8 and 2
 7 and 3
 6 and 4
 5 and 5

5 is its own complement.

The term "Friends of Ten" is commonly used in educational circles. In Circlemaths this concept is taken much further, we need "Friends of Nine" and "Friends of Eight" and so on indefinitely. For this reason and also because the term "complement" implies an amount required to complete up to a full circle, we prefer to call these linked numbers "complements" instead.

Hence the name of the method. The "Add a Complement" method.

From the simple idea of picking up a ten and putting back a one to subtract nine (for example), a general method when a subtraction goes Below Zero can be produced:

Step 1: Spot it goes below zero for a start

Step 2: Reduce the tens digit by 1

Step 3: Add the complement of the number you are taking away to the units column.

This is easy to accomplish and henceforth can be done with great rapidity.

Examples 1:

$$\begin{array}{r} 73 \\ - 9 \\ \hline 64 \end{array}$$

Step 1: 3 - 9 goes Below Zero. This "won't go", is "hard" or class B
 Step 2: Drop from seventy down to sixty
 Step 3: The complement of 9 is 1. Add 1 to 3 to get 4.

Examples 2:

$$\begin{array}{r} 47 \\ - 9 \\ \hline 38 \end{array}$$

Step 1: 7 - 9 goes Below Zero. This "won't go", is "hard" or class B
Step 2: Drop from forty down to thirty.
Step 3: The complement of 9 is 1. Add 1 to 7 to get 8.

Examples 3:

$$\begin{array}{r} 34 \\ - 8 \\ \hline 26 \end{array}$$

Step 1: 4 - 8 goes Below Zero. This "won't go", is "hard" or class B
Step 2: Drop from thirty down to twenty.
Step 3: The complement of 8 is 2. Add 2 to 4 to get 6.

Repeat Subtraction of 9:

$$\begin{array}{r} 75 \\ - 9 \\ \hline 66 \\ 57 \\ 48 \\ 39 \\ 30 \\ 21 \\ 12 \\ 03 \end{array}$$

Step 1: 4 - 8 goes Below Zero. This "won't go", is "hard" or class B
Step 2: Drop from seventy down to sixty.
Step 3: The complement of 9 is 1. Add 1 to 5 to get 6.
Drop from sixty to fifty. Add 1 to 6 to get 7.
Drop from fifty to forty. Add 1 to 7 to get 8.
Drop from forty to thirty. Add 1 to 8 to get 9.
Take 9 off 39 to leave thirty exactly.
Drop from thirty to twenty. Add 1 to 0 to get 1.
Drop from twenty to ten. Add 1 to 1 to get 2.
Drop from ten to zero tens. Add 1 to 2 to get 3.

- With practice that can be done as quick as you can write the answers up.
- We haven't mentioned step 1 each time, which is to check if it will go or not.
- At 39 - 9 the subtraction will go. It is class A. We simply subtract from the units column.