

Lesson 3

Why the "Add a Complement" Method Works

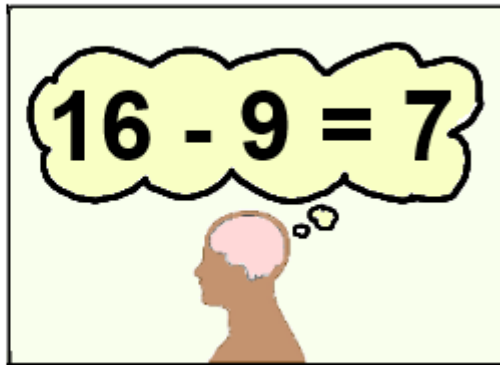
This is the quick guide to the video. For more complete details watch video 3.

Goals:

- To show WHY the "Add a Complement" method works
 - To relate it to other methods
 - To see the maths beneath it

Traditional Technique

$$\begin{array}{r} 3 \quad 1 \\ \cancel{4}6 \\ - 9 \\ \hline \end{array}$$



We "borrow" a ten from the tens column and "add it to the units" column.

This makes the 6 become a 16.

Now we are faced with solving $16 - 9 = 7$ which **will** go.

The difficulty here is that we need to call upon our memory. We need to have memorized $16 - 9 = 7$.

If the sum were $45 - 9$ we would need to know " $15 - 9 = 6$ ".

If it were $44 - 9$ we would need " $14 - 9 = 5$ " and so on.

Add to that list if we wanted to subtract 8 instead of 9.

For $16 - 8$ we would need to know $16 - 8 = 8$.

For $15 - 8$ we would need $15 - 8 = 7$.

And so on.

I think you get the point. We are placing a great load on our memory this way.

It might be countered:

"But these are just the flip sides of addition sums like $9 + 5 = 14$ which we need to have memorized anyway".

However:

1. Just because you have memorized the addition $9 + 5 = 14$ does not mean you can retrieve the associated subtraction that $14 - 9$ comes to 5
2. We have not shown Circlemath addition yet. With addition in Circlemath there is no more need to memorize these facts for addition than there is for the subtraction.

Notice furthermore that the traditional method produces an answer in reverse order. Here is an example of a long subtraction half way through using the this method:

$$\begin{array}{r} 7981352 \\ - \underline{1547861} \\ 3491 \end{array}$$

The answer typically comes out "back to front", with the least significant digits coming first. In Circlemaths we usually choose to reverse this order (although we could equally keep the same order, but why would you want to?).

The Add A Complement Method

$$\begin{array}{r} 46 \\ -9 \\ \hline \end{array}$$



Essentially it is the same strategy, except we:

1. Reverse the order. The moment we see that $6 - 9$ "won't go" we drop from forty to thirty. Our answer is half spoken at this point. "Thirty..."
2. Intercept the "borrowed" \$10 bill instead of adding it to the 6 to get 16. We see it as complements, as "9 and 1". If we take away the 9 that leaves the 1 we are adding to the units digit (as shown above).

Doing it this way means we don't need to know $16 - 9 = 7$. We just need our single picture of the complements and how to add numbers below 10 (such as $6 + 1 = 7$).

The Algebra Beneath It

The traditional method follows this path:

$$\begin{aligned}
 &46 - 9 \\
 &= (40 + 6) - 9 \\
 &= (30 + 10 + 6) - 9 \\
 &= (30 + 16) - 9 && // \text{ a quite unnecessary addition step here} \\
 &= 30 + (16 - 9) && // \text{ resulting in needing to know "16 - 9" as a cold fact} \\
 &= 30 + 7 \\
 &= 37
 \end{aligned}$$

The Circlemaths "Add a Complement" method follows this path instead:

$$\begin{aligned}
 &46 - 9 \\
 &= (40 + 6) - 9 \\
 &= (30 + 10 + 6) - 9 \\
 &= 30 + (10 - 9) + 6 && // \text{ 10 - 9 produces the complement of 9 in 10-circle} \\
 &= 30 + 1 + 6 \\
 &= 30 + (1 + 6) && // \text{ 1 + 6 is an addition below 10} \\
 &= 30 + 7 \\
 &= 37
 \end{aligned}$$

The Add a Complement method also calls upon the memory, however its call is nowhere near as great. A child only needs to know the complements in 10-circle (see picture above) and those numbers which add to a total below 10.

This is much less than needing to know the list of subtractions such as $16 - 9 = 7$ etc. part of which is pictured below:

$\overset{1}{0}$	$\overset{1}{1}$	$\overset{1}{2}$	$\overset{1}{3}$	$\overset{1}{4}$	$\overset{1}{5}$	$\overset{1}{6}$	$\overset{1}{7}$	$\overset{1}{8}$
$\overset{1}{-9}$	$\overset{1}{-9}$	$\overset{1}{-9}$	$\overset{1}{-9}$	$\overset{1}{-9}$	$\overset{1}{-9}$	$\overset{1}{-9}$	$\overset{1}{-9}$	$\overset{1}{-9}$
$\overset{1}{1}$	$\overset{1}{2}$	$\overset{1}{3}$	$\overset{1}{4}$	$\overset{1}{5}$	$\overset{1}{6}$	$\overset{1}{7}$	$\overset{1}{8}$	$\overset{1}{9}$
$\overset{1}{0}$	$\overset{1}{1}$	$\overset{1}{2}$	$\overset{1}{3}$	$\overset{1}{4}$	$\overset{1}{5}$	$\overset{1}{6}$	$\overset{1}{7}$	
$\overset{1}{-8}$	$\overset{1}{-8}$	$\overset{1}{-8}$	$\overset{1}{-8}$	$\overset{1}{-8}$	$\overset{1}{-8}$	$\overset{1}{-8}$	$\overset{1}{-8}$	
$\overset{1}{2}$	$\overset{1}{3}$	$\overset{1}{4}$	$\overset{1}{5}$	$\overset{1}{6}$	$\overset{1}{7}$	$\overset{1}{8}$	$\overset{1}{9}$	
$\overset{1}{0}$	$\overset{1}{1}$	$\overset{1}{2}$	$\overset{1}{3}$	$\overset{1}{4}$	$\overset{1}{5}$	$\overset{1}{6}$		
$\overset{1}{-7}$	$\overset{1}{-7}$	$\overset{1}{-7}$	$\overset{1}{-7}$	$\overset{1}{-7}$	$\overset{1}{-7}$	$\overset{1}{-7}$		
$\overset{1}{3}$	$\overset{1}{4}$	$\overset{1}{5}$	$\overset{1}{6}$	$\overset{1}{7}$	$\overset{1}{8}$	$\overset{1}{9}$		
$\overset{1}{0}$	$\overset{1}{1}$	$\overset{1}{2}$	$\overset{1}{3}$	$\overset{1}{4}$	$\overset{1}{5}$			
$\overset{1}{-6}$	$\overset{1}{-6}$	$\overset{1}{-6}$	$\overset{1}{-6}$	$\overset{1}{-6}$	$\overset{1}{-6}$			
$\overset{1}{4}$	$\overset{1}{5}$	$\overset{1}{6}$	$\overset{1}{7}$	$\overset{1}{8}$	$\overset{1}{9}$			

$$\begin{array}{r}
 \cancel{46} \\
 -\cancel{9} \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 \cancel{47} \\
 -\cancel{9} \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 \cancel{48} \\
 -\cancel{9} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \cancel{46} \\
 -\cancel{8} \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 \cancel{47} \\
 -\cancel{8} \\
 \hline
 \end{array}$$

