Lesson 7

Why Long Division Works

This is the quick guide to the video. For more complete details watch video 7.

Goals:

To see how repeated subtraction is the same thing as long division

To see what a remainder means

The Link between Repeat Subtraction and Long Division:

Let us suppose that some primitive hunters have made a kill and want to divide it out equally among themselves.
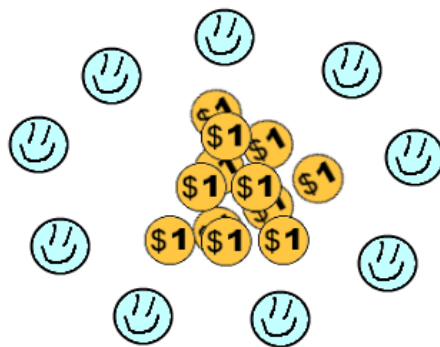
Or perhaps our primitive hunters are slightly more modern and have made a killing on the stock market,

so we can use dollars to make sense of long division!

So suppose we have $36 and we want to divide it up amongst 9 people evenly.

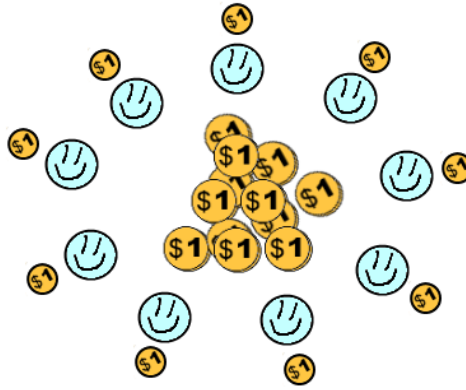How could we go about it if we didn't know our tables or have a calculator?

Our start position, $36 and 9 people to divide it between.

Let each of our hunters reach in and grab one gold coin each from the pile:

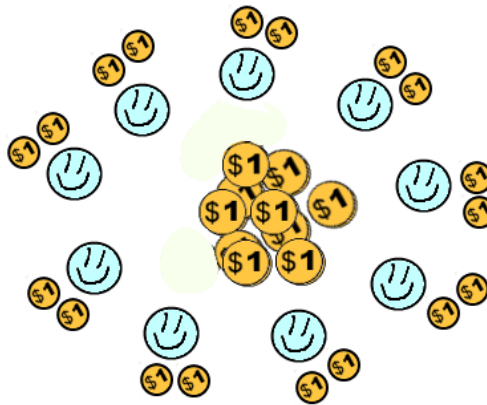$$9\overline{)36}$$
$$27$$

The pile has been reduced by 9 gold coins in all.
So we subtract 9 off the 36 to get 27 coins in the pile.

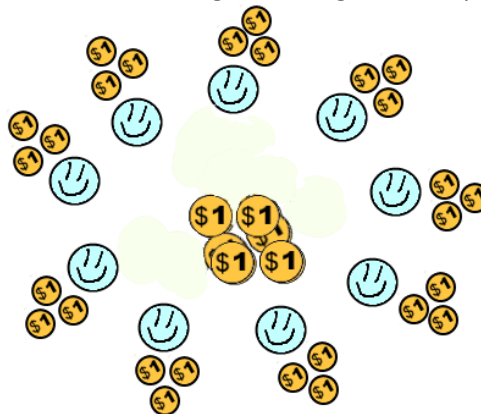Do it again. Each hunter reaches in and grabs another gold coin each from the pile:

$$9\overline{)36}$$
$$27$$
$$18$$

The pile has once again been reduced by 9 gold coins in all.
So we subtract 9 off 27 to get 18 coins now left in the pile.
They can do it again…

Once more each hunter reaches in and grabs one gold coin apiece:

$$9\overline{)36}$$
$$27$$
$$18$$
$$09$$

Each hunter now has 3 gold coins.
They have reached in 3 times.
The pile has been reduced by another 9 coins in all.
We subtract 9 off the 18 that were there to see we have just 9 gold coins left.
Can the hunters do it again?
You bet!

For a fourth and final time the hunters reach in and grab a gold coin each:

```
9 )3 6
   2 7
   1 8
   0 9
   0 0
```

Each hunter now has 4 gold coins.
They have reached in 4 times.
The pile has been reduced by its final 9 coins down to zero.

We ask when we divide 36 by 9
"How many times will 9 go into 36?"

We have shown how many times by actually taking them out time after time.
9 will "go into" (or come out of!) 36 a total of 4 times.
The number of times each hunter reached in.
The same number of coins each hunter has as a result.

```
         4
  9 )3 6
    2 7
    1 8
4 steps ← 0 9
    0 0
```
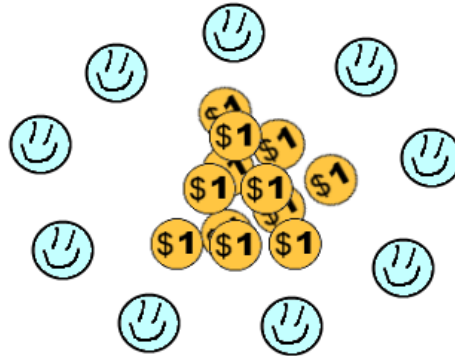
As you can see counting up the number of subtraction steps
tells you how many 9's came out of 36
which is the same as
how many 9's are in 36

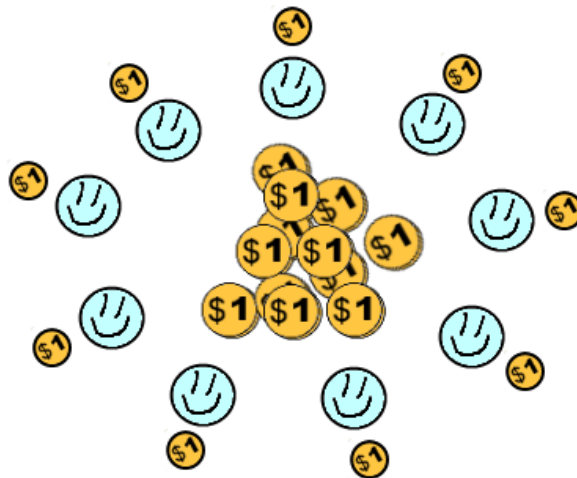Repeated subtraction is the same as long division.

The Remainder

Suppose we repeat the above exercise. But we throw in one more gold coin to the pot to start with.
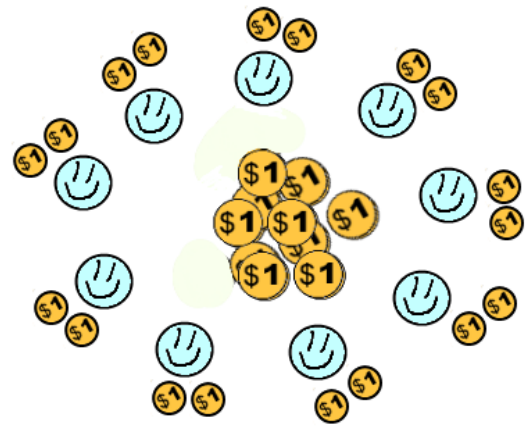This time we start with $37.

$$9\overline{)37}$$

We begin with 37 gold coins to be divided between 9 people, as above.
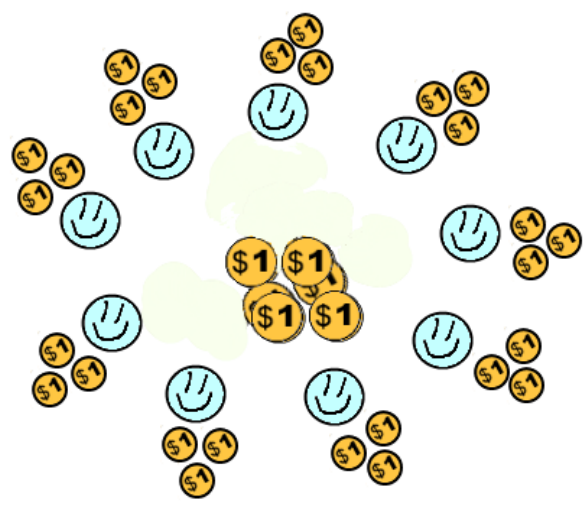
$$9\overline{)37} \atop 28$$

Each person grabs one gold coin.
That's 9 gold coins in all.
So the remaining pile has 37 - 9 = 28 gold coins.

```
  9 )3 7
    2 8
    1 9
```

Repeat the process.
Each person now has 2 gold coins.
The remaining pile has 28 - 9 = 19 gold coins.

```
  9 )3 7
    2 8
    1 9
    1 0
```

A third time and another 9 gold coins get taken from the pile.
Leaving 19 - 9 = 10 coins in total.

```
   4  r 1
9 )3 7
   2 8
   1 9
   1 0
   0 1
```
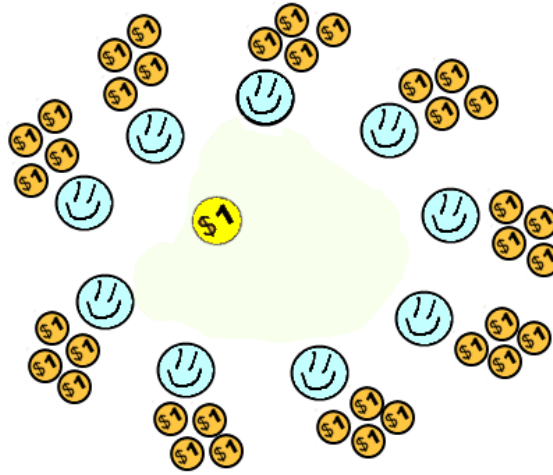
The fourth and final time each person takes a coin.
This leaves 10 - 9 = 1 gold coin remainder.
The group took out 1 gold coin each a total of 4 times.
Each person has 4 gold coins.
There is 1 left over.
That is the remainder and that is all it means.
One which has yet to be divided.
To divide it out we would need to break it into parts.

Fractions

If we get out a magnifying glass we might be able to make that single gold coin left over look a bit bigger.
Then we might be able to break it into 9 equal parts that can be divided out.
Here goes:

```
   4  r 1
9 )3 7
   2 8
   1 9
   1 0
   0 1
```
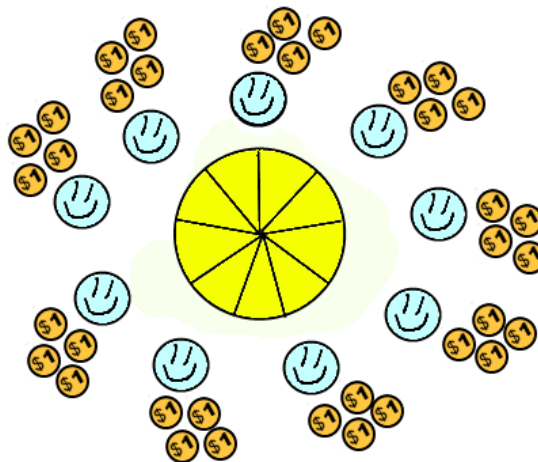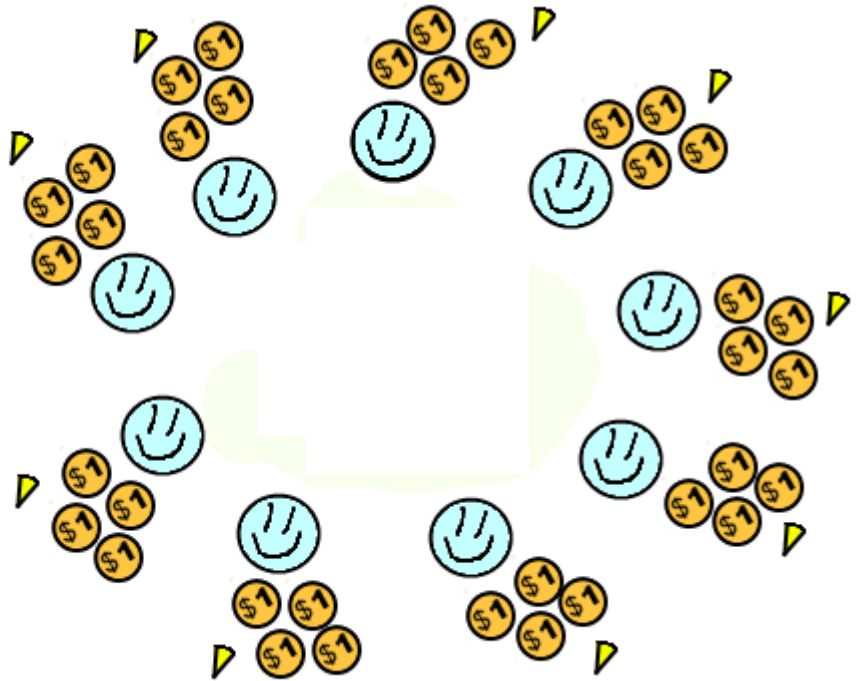
The 1 remainder gold coin in the middle
magnified - not to scale!
and divided into 9 equal parts.

We could now distribute the 1 remainder as ninths to each person.
That is, they could each grab one of those parts

so they would have 1/9th of a part each:

```
        4 r 1
9 )3 7
   2 8
   1 9
   1 0
   0 1
```

```
        4 ¹/₉
9 )3 7
```

The answer is otherwise expressed as 4 1/9 meaning each person gets 4 coins and 1/9 of a coin.