

## Lesson 5

### Subtract Any Which Way Series

#### Long Subtraction: When BOTH are HARD

*This is the quick guide to the video. For more complete details watch "Subtract Any Which Way" video 5.*

#### Goal:

To introduce Long Subtraction so it can be done in any direction with ease.  
To show how to handle a subtraction which goes BELOW zero (and is "hard")  
and whose NEIGHBOUR  
is ALSO BELOW zero (and "hard").

#### The Hard-Hard Steps

$$\begin{array}{r} 5613981706 \\ - 2781635912 \\ \hline \end{array}$$

In the above subtraction the subtraction pairs which come to a result  
BELOW ZERO  
have been highlighted.

We don't know what a subtraction pair like "1 - 8" comes to  
so we can also term it "hard".

In order to work it out  
we will need  
to use one  
of our  
strategies.

We have TWO strategies:

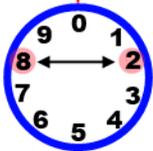
Add a Complement

Complement of the Difference

$$\begin{array}{r} 5613981706 \\ - 2781635912 \\ \hline \end{array}$$

Because 8 is one of the relatively "big" numbers

I'll use the Add a Complement strategy here:

$$\begin{array}{r} 5613981706 \\ - 2781635912 \\ \hline \end{array}$$


Add a Complement

Add the complement of 8 (which is 2) to the 1:

$$1 - 8 = 1 + 2 = 3$$

in 10-circle.

So 3

is "the answer".

But is it?

We also need to LOOK AHEAD to the RIGHT

to check the NEIGHBOUR.

In this case the neighbour is

$$3 - 1$$

which is "Above Zero" (easy).

It can be worked out  
without needing to call at all  
on the "1 - 8" subtraction pair.

It has no effect.

The final answer remains 3.

How about the "5 - 6" subtraction pair?

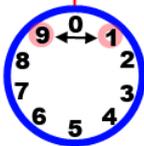
$$\begin{array}{r} 5 \ 6 \ 1 \ 3 \ 9 \ 8 \ 1 \ 7 \ 0 \ 6 \\ - 2 \ 7 \ 8 \ 1 \ 6 \ 3 \ 5 \ 9 \ 1 \ 2 \\ \hline ? \ 3 \end{array}$$

Again you can't take 7 off 6 without it going Below zero.

It's "hard" so we need to use one of our two strategies.

In this case we will choose to use the

Complement of the Difference Strategy:

$$\begin{array}{r} 5 \ 6 \ 1 \ 3 \ 9 \ 8 \ 1 \ 7 \ 0 \ 6 \\ - 2 \ 7 \ 8 \ 1 \ 6 \ 3 \ 5 \ 9 \ 1 \ 2 \\ \hline ? \ 3 \end{array}$$


Complement of Difference

Either strategy will work,

but this one is better because it's obvious that

6 and 7 are just 1 apart.

The difference is 1

The complement of that is 9.

The answer is 9.

But is it?

Again we must always LOOK AHEAD

to the NEIGHBOUR on the RIGHT:

**LOOK →**

$$\begin{array}{r} 5 \ 6 \ 1 \ 3 \ 9 \ 8 \ 1 \ 7 \ 0 \ 6 \\ - 2 \ 7 \ 8 \ 1 \ 6 \ 3 \ 5 \ 9 \ 1 \ 2 \\ \hline \end{array}$$

? 3

**Above zero**



**Below zero**



In this case the right hand neighbour is  
the subtraction pair "1 - 8".

We don't need to know what it comes to  
just that it comes to a result  
BELOW ZERO.

It WILL need to borrow from the 6-7 pair.

The end result of the 6-7 pair

WILL

GO DOWN BY ONE.

So

6 - 7 = 9 in 10-circle is NOT the final answer

We reduce it from 9 down to 8:

**LOOK →**

$$\begin{array}{r} 5 \ 6 \ 1 \ 3 \ 9 \ 8 \ 1 \ 7 \ 0 \ 6 \\ - 2 \ 7 \ 8 \ 1 \ 6 \ 3 \ 5 \ 9 \ 1 \ 2 \\ \hline 9 \ 3 \end{array}$$

~~9~~  
**8**



The General Approach  
when a subtraction pair goes  
BELOW ZERO  
is:

Use of the two strategies to find the answer

Look at its neighbour (on the right)

If it's BELOW zero

REDUCE the answer by ONE

If its ABOVE zero

LEAVE IT AS IT IS.

The rest of the hard values can be worked out:

$$\begin{array}{r} 5 \ 6 \ 1 \ 3 \ 9 \ 8 \ 1 \ 7 \ 0 \ 6 \\ - 2 \ 7 \ 8 \ 1 \ 6 \ 3 \ 5 \ 9 \ 1 \ 2 \\ \hline 9 \ 3 \end{array}$$

~~9~~   ~~3~~   ~~6~~   ~~8~~   ~~9~~  
**8**   **5**   **7**

$$1 - 5 = 1 + 5 = 6 \text{ (Add a Complement)}$$

$$7 - 9 = 7 + 1 = 8 \text{ (Add a Complement)}$$

0 - 1 = Comp of 1 = 9 (Complement of Difference)

and

we reduced the 6 to 5 because the neighbour of 1-5 is 7-9 which is below zero

and the 8 to a 7 because the neighbour of 7-9 is 0-1 which is below zero.

**Question:** What would happen if the very end subtraction pair came to a result BELOW ZERO.

There is no neighbour on the right. So would you reduce it or not?

**Answer:** Leave it as it is. There is nothing on its right so nothing will be effecting it.

**Question:** What about the remaining digits of the answer?

**Answer:** Just fill them in using the information from the previous video's.

They are easy. "5-2 = 3" for instance, but reduces down to 2

because its neighbour is "6-7" which is Below Zero.

"6-2" on the end is just 4 and stays that way

because it has no neighbour.

**Question:** Do we have to subtract in that order?

**Answer:** Absolutely not!

You can now subtract in any order whatsoever you like.

Forward to back, back to front, upside down - it doesn't matter!

However

It is more sensible to subtract

LEFT TO RIGHT

so your answer reads correctly as it emerges.