

## Lesson 7

### Subtract Any Which Way Series

#### Zero Exactly

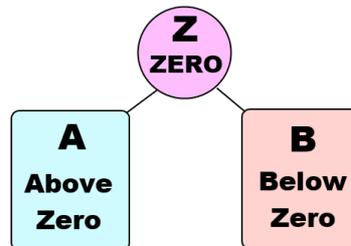
*This is the quick guide to the video. For more complete details watch "Subtract Any Which Way" video 7.*

#### Goal:

To show what happens when  
a subtraction pair  
subtracts to ZERO exactly.

#### Review

When a subtraction pair subtracts  
to ZERO EXACTLY  
it is neither "Above Zero" nor "Below Zero".



It turns out it can flip between the two.

Consider the following subtractions:

$$\begin{array}{r} 8004 \\ - 1002 \\ \hline 7002 \end{array}$$

$$\begin{array}{r} 8002 \\ - 1004 \\ \hline 6998 \end{array}$$

The only difference is the subtraction pair on the end.

If the subtraction pair on the end is ABOVE ZERO (as in  $4-2 = 2$ )

The answer is easy and goes straight down (2 in this case)

The bit on the left ( $800 - 100 = 700$ ) goes down as it is.

If the subtraction pair on the end is BELOW ZERO (as in  $2-4$ )

The answer is hard and we need to use one of our strategies to get the answer (8 in this case)

The bit on the left ( $800 - 100 = 700$ ) is REDUCED BY ONE to 699.

Effectively

the 0's turn into 9's

the digit at the very front is reduced by one (from 7 to 6)

The behaviour of the subtraction pairs coming to ZERO is determined by the subtraction pair on the end of them.

## The Sandwich

Think of the block of zeros as being the filling in a sandwich.

Two bits of bread are on either end.

In the example below the far end subtraction pair ( $6 - 4$ )

comes to a result ABOVE ZERO.

It is easy to work out ( $6-4 = 2$ )

and it does not effect the block of zeros in any way.

Nor do the zeros effect the other slice of bread (the  $9 - 1$ ).

$$\begin{array}{r}
 8954673986 \\
 -4154673984 \\
 \hline
 800000002
 \end{array}$$

There is no impact either on the  $8-4 = 4$  at the start of the subtraction.

Now we turn the last digits about.

We reverse the end digits.

Instead of  $6 - 4 = 2$  (ABOVE zero)

we have

$4 - 6 = ?$  (BELOW zero):

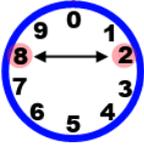
$$\begin{array}{r}
 8954673984 \\
 -4154673986 \\
 \hline
 00000000
 \end{array}$$

First off we don't know what  $4 - 6$  comes to.

It is "hard". We will need to use a strategy:

$$\begin{array}{r}
 8954673984 \\
 -4154673986 \\
 \hline
 00000008
 \end{array}$$

Compliment of Difference



That's easy to do.

The difference between 4 and 6 is 2

The complement of that difference is 8

So 8 is our answer.

There is no neighbour on its right

So nothing to alter it

It remains 8.

But now look at the 8 - 8 subtraction pair which came to zero.

Its neighbour is the 4 - 6 = 8 pair

which is BELOW ZERO.

And when the neighbour on the right is

BELOW ZERO

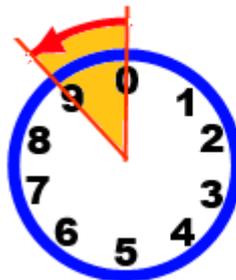
we need to DECREASE THE ANSWER BY ONE.

The answer was ZERO ( $8 - 8 = 0$ ).

What is 1 less than 0?

We can see it in the 10-circle:

**0 - 1 = 9**  
**in 10-cir**



So our answer "reduces" to "-1" which is 9 in 10-circle.

(Don't forget we are always winding around 10-circle).

### **The Cascading Effect**

But this effect now cascades through all the zeros.

To see why examine what has just happend:

Taking 6 off 4 can't be done.

We needed to "borrow" from the neighbour on the left.

That's the top 8 in the 8 - 8 subtraction pair.

*So it must have gone down by one to 7:*

$$\begin{array}{r}
 8954673984 \\
 -4154673986 \\
 \hline
 00000098
 \end{array}$$

The diagram shows a subtraction problem with a horizontal line. The top row contains the digits 8, 9, 5, 4, 6, 7, 3, 9, 8, 4. The bottom row contains the digits -4, 1, 5, 4, 6, 7, 3, 9, 8, 6. A horizontal line is drawn below the bottom row. Below the line, the digits 0, 0, 0, 0, 0, 0, 9, 8 are written. A red box highlights the first two digits (8 and 9) of the top row. A blue box highlights the digits 5, 4, 6, 7, 3, 9, 8 of the top row. A red box highlights the digits 9 and 8 of the bottom row. A red box highlights the digits 9 and 8 of the result row. A red arrow points from the 9 in the top row to the 8 in the bottom row. A blue arrow points from the 4 in the top row to the 6 in the bottom row.

But that means the subtraction pair  $9 - 9 = 0$

now has the neighbour  $7 - 8 = 9$

and " $7 - 8 = 9$ " is BELOW ZERO.

$$\begin{array}{r}
 8954673984 \\
 -4154673986 \\
 \hline
 000000998
 \end{array}$$

The diagram shows the same subtraction problem as above, but with a red box highlighting the digits 9 and 9 in the result row. A red box highlights the digits 9 and 8 in the bottom row. A red box highlights the digits 9 and 8 in the top row. A red arrow points from the 9 in the top row to the 8 in the bottom row. A blue arrow points from the 4 in the top row to the 6 in the bottom row. A red box highlights the digits 9 and 9 in the result row.

Which means of course that the " $9 - 9 = 0$ " needs to be REDUCED by ONE in turn.

It reduces from 0 down by 1 (around the 10-circle, see above picture of circle) to 9 as well.

And that happens of course because the  $7 - 8 = 9$

had to borrow from the  $9 - 9 = 0$ .

Which meant the top 9 of " $9 - 9$ " had to drop to an 8

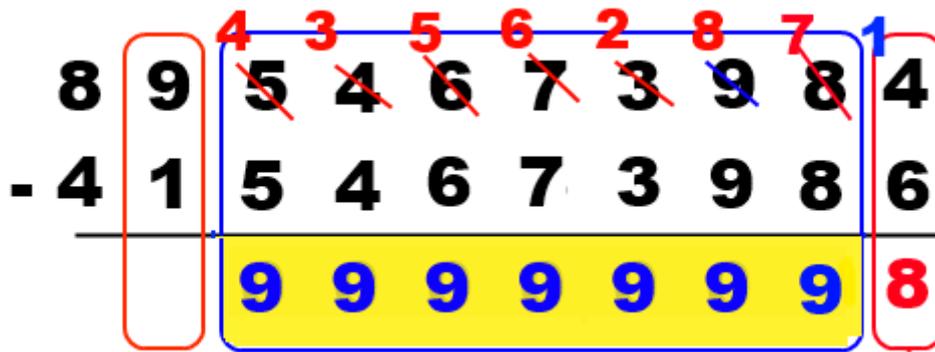
which makes it no longer  $9 - 9 = 0$

but  $8 - 9 = 9$

which in turn means it also has gone

BELOW ZERO

and so on and on it goes, cascading through the line of zeros:



All the zeros turn into 9's.

When we get to the other end of the sandwich (the 9 - 1)

we can see  $9 - 1 = 8$

BUT

its neighbour is not  $5 - 5 = 0$  anymore

but

$4 - 5 = 9$

a subtraction BELOW ZERO.

So the front of the sandwich is effected ("infected") by the

cascading zeros also.

It must drop by 1 to become not 8 but 7.

That is the end of the effect.

This buffers the effect and stops it propogating further.

The  $8 - 4 = 4$  just goes down in the usual manner:

$$\begin{array}{r}
 89\cancel{5}\cancel{4}\cancel{6}\cancel{7}\cancel{3}\cancel{9}\cancel{8}4 \\
 -4154673986 \\
 \hline
 479999998
 \end{array}$$

The diagram shows a subtraction problem with a 'sandwich' of numbers. The top row is 8954673984, the middle row is 4154673986, and the bottom row is 479999998. A red box encloses the first and last digits of each row. A blue box encloses the digits 5, 4, 6, 7, 3, 9, 8 in the top row and 9, 9, 9, 9, 9, 9, 9 in the bottom row. Red numbers 4, 3, 5, 6, 2, 8, 7, 1 are written above the top row, and a blue number 1 is written above the last digit of the top row. Red diagonal lines cross out the digits 5, 4, 6, 7, 3, 9, 8 in the top row.

**Question:** What would have happened if the sandwich started with 1 - 9 instead of 9 - 1?

**Answer:** We would have needed to use one of our strategies, probably Add a Complement.

Then  $1 - 9 = 1 + 1 = 2$  in 10-circle.

The answer would appear to be 2.

BUT as above

the neighbour is now  $4 - 5 = 9$  instead of  $5 - 5 = 0$ .

It is really going "BELOW ZERO"

so the final answer would not be 2

It would need to drop down by one

to a 1.

Then the  $8 - 4 = 4$  would also need to drop to a 3

as its neighbour, the  $1 - 9$ , would have been "BELOW ZERO".

The answer would then be:

3 7 9 9 9 9 9 9 8

*This concludes the series*

*on Long Subtraction.*

*You should now be able to subtract*

*in any order whatsoever*

*easily.*

