Lesson 7

Subtract Any Which Way Series

Zero Exactly

This is the quick guide to the video. For more complete details watch "Subtract Any Which Way" video 7.

Goal:

To show what happens when

a subtraction pair

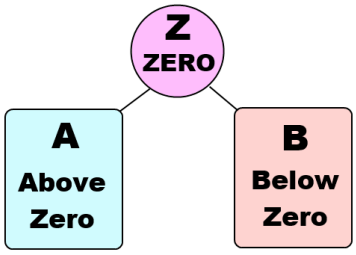
subtracts to ZERO exactly.

Review

When a subtraction pair subtracts

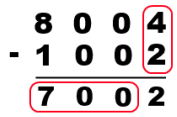
to ZERO EXACTLY

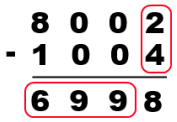
it is neither "Above Zero" nor "Below Zero".



It turns out it can flip between the two.

Consider the following subtractions:





The only difference is the subtraction pair on the end.

If the subtraction pair on the end is ABOVE ZERO (as in 4-2 = 2)

The answer is easy and goes straight down (2 in this case)

The bit on the left (800 - 100 = 700) goes down as it is.

If the subtraction pair on the end is BELOW ZERO (as in 2-4)

The answer is hard and we need to use one of our strategies to get the answer (8 in this case)

The bit on the left (800 - 100 = 700) is REDUCED BY ONE to 699.

Effectively

the 0's turn into 9's

the digit at the very front is reduced by one (from 7 to 6)

The behaviour of the subtraction pairs coming to ZERO

is determined by the subtraction pair on the end of them.

The Sandwich

Think of the block of zeros as being the filling in a sandwich.

Two bits of bread are on either end.

In the example below the far end subtraction pair (6 - 4)

comes to a result ABOVE ZERO.

It is easy to work out (6-4 = 2)

and it does not effect the block of zeros in any way.

Nor do the zeros effect the other slice of bread (the 9 - 1).

```
 8 9 5 4 6 7 3 9 8 6
-4 1 5 4 6 7 3 9 8 4
-------------------
 8 0 0 0 0 0 0 0 2
```

There is no impact either on the 8-4 = 4 at the start of the subtraction.

Now we turn the last digits about.

We reverse the end digits.

Instead of 6 - 4 = 2 (ABOVE zero)

we have

4 - 6 = ? (BELOW zero):

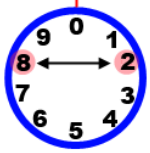
```
 8 9 5 4 6 7 3 9 8 4
-4 1 5 4 6 7 3 9 8 6
-------------------
   0 0 0 0 0 0 0
```

First off we don't know what 4 - 6 comes to.

It is "hard". We will need to use a strategy:

```
 8 9 5 4 6 7 3 9 8 4
-4 1 5 4 6 7 3 9 8 6
-------------------
   0 0 0 0 0 0 0 8
```

Compliment of Difference



That's easy to do.

The difference between 4 and 6 is 2

The complement of that difference is 8

So 8 is our answer.

There is no neighbour on its right

So nothing to alter it

It remains 8.

But now look at the 8 - 8 subtraction pair which came to zero.

Its neighbour is the 4 - 6 = 8 pair

which is BELOW ZERO.

And when the neighbour on the right is

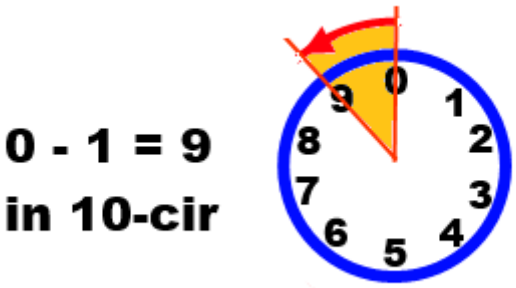
BELOW ZERO

we need to DECREASE THE ANSWER BY ONE.

The answer was ZERO (8 - 8 = 0).

What is 1 less than 0?

We can see it in the 10-circle:



So our answer "reduces" to "-1" which is 9 in 10-circle.

(Don't forget we are always winding around 10-circle).

The Cascading Effect

But this effect now cascades through all the zeros.

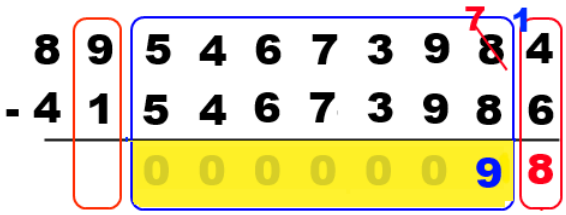
To see why examine what has just happend:

Taking 6 off 4 can't be done.

We needed to "borrow" from the neighbour on the left.

That's the top 8 in the 8 - 8 subtraction pair.

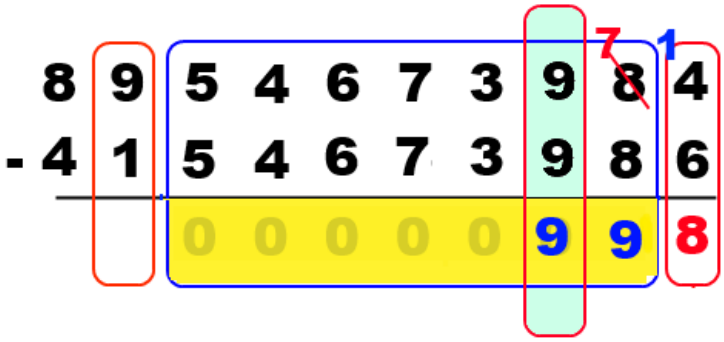
So it must have gone down by one to 7:



But that means the subtraction pair 9 - 9 = 0

now has the neighbour **7** - 8 = 9

and "7 - 8 = 9" is BELOW ZERO.



Which means of course that the "9 -9 = 0" needs to be REDUCED by ONE in turn.

It reduces from 0 down by 1 (around the 10-circle, see above picture of circle) to 9 as well.

And that happens of course because the 7 - 8 = 9

had to borrow from the 9 - 9 = 0.

Which meant the top 9 of "9 - 9" had to drop to an 8

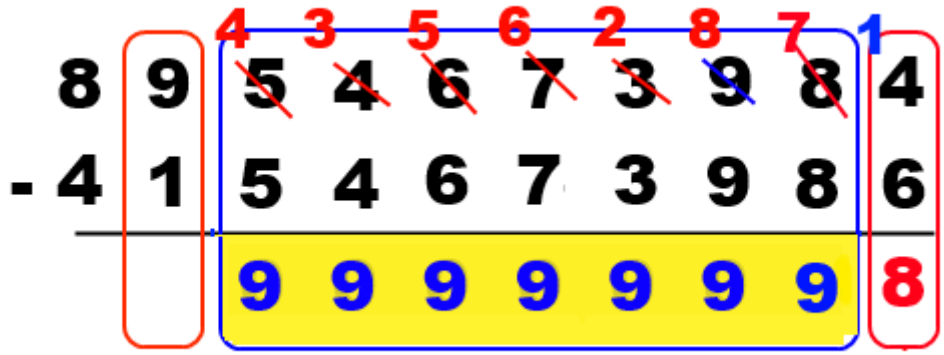
which makes it no longer 9 - 9 = 0

but 8 - 9 = 9

which in turn means it also has gone

BELOW ZERO

and so on and on it goes, cascading through the line of zeros:



All the zeros turn into 9's.

When we get to the other end of the sandwich (the 9 - 1)

we can see 9 - 1 = 8

BUT

its neighbour is not 5 - 5 = 0 anymore

but

4 - 5 = 9

a subtraction BELOW ZERO.

So the front of the sandwich is effected ("infected") by the

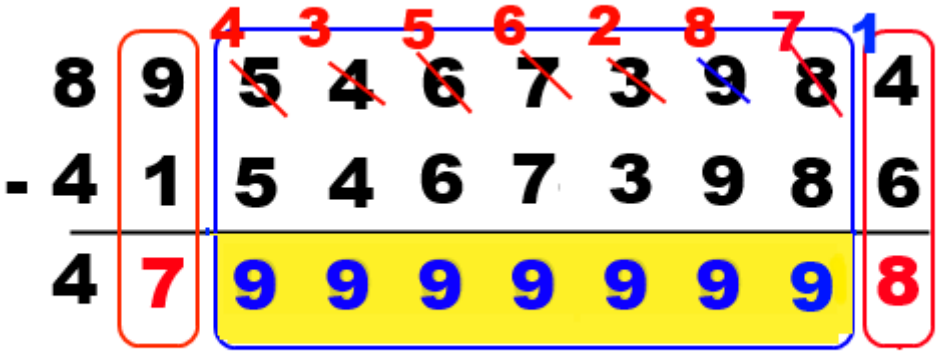
cascading zeros also.

It must drop by 1 to become not 8 but 7.

That is the end of the effect.

This buffers the effect and stops it propogating further.

The 8 - 4 = 4 just goes down in the usual manner:



Question: What would have happened if the sandwich started with 1 - 9 instead of 9 - 1?

Answer: We would have needed to use one of our strategies, probably Add a Complement.

Then 1 - 9 = 1 + 1 = 2 in 10-circle.

The answer would appear to be 2.

BUT as above

the neighbour is now 4 - 5 = 9 instead of 5 - 5 = 0.

It is really going "BELOW ZERO"

so the final answer would not be 2

It would need to drop down by one

to a 1.

Then the 8 - 4 = 4 would also need to drop to a 3

as its neighbour, the 1 - 9, would have been "BELOW ZERO".

The asnwer would then be:

3 7 9 9 9 9 9 9 9 8

This concludes the series

on Long Subtraction.

You should now be able to subtract

in any order whatsoever

easily.