Lesson 9

Subtract Any Which Way Series

Memorizing the 9-circle Results

This is the quick guide to the video. For more complete details watch "Subtract Any Which Way" video 9.

Goal:

To show that learning the 9-circle subtractions

is not all that difficult (we've already seen its useful).

Why We Want to Know the 9-circle Subtractions

This was covered in the previous video/webpage. Just as a reminder:

If a subtraction pair is "Hard-Hard" then we can
work the subtraction out in 9-circle
and that's it.
No need to "reduce the result by 1"
This saves "double handling" and makes subtraction
faster and easier.

Example:

$$\begin{array}{r} 6\ 0\ 5\ 7\ 3\ 4 \\ -\ 2\ 1\ 6\ 8\ 4\ 5 \\ \hline 3\ 8\ 8\ 8\ 8\ 9 \end{array}$$

The subtractions in red go below zero (we could call them "Hard")

and their right hand neighbours also go below zero (and are "Hard")

They are all "Hard-Hard" type subtractions.

We just need to subtract them in 9-circle:

The difference between them is 1

(1 and 0 are 1-apart, 5 and 6 are 1-apart etc.)

The complement of that difference in 9-circle is 8.

They all come to 8.

No reduction necessary.

The list of 9-circle Results to Memorize

Here is the total list - (enough to put anyone off!)

-	0	1	2	3	4	5	6	7	8
0	0	8	7	6	5	4	3	2	1
1	1	0	8	7	6	5	4	3	2
2	2	1	0	8	7	6	5	4	3
3	3	2	1	0	8	7	6	5	4
4	4	3	2	1	0	8	7	6	5
5	5	4	3	2	1	0	8	7	6
6	6	5	4	3	2	1	0	8	7
7	7	6	5	4	3	2	1	0	8
8	8	7	6	5	4	3	2	1	0

The good news:
Most of them don't need to be memorized.

Read them left to right (row to column).
So for example this shows "2 - 1 = 1":

-	0	1
0	0	8
1	1	0
2	2	1

You can see we can eliminate all results that subtract to zero or above such as 1-1=0 and 2-1 = 1 etc.
This leaves:

-	0	1	2	3	4	5	6	7	8
0		8	7	6	5	4	3	2	1
1			8	7	6	5	4	3	2
2				8	7	6	5	4	3
3					8	7	6	5	4
4						8	7	6	5
5							8	7	6
6								8	7
7									8
8									

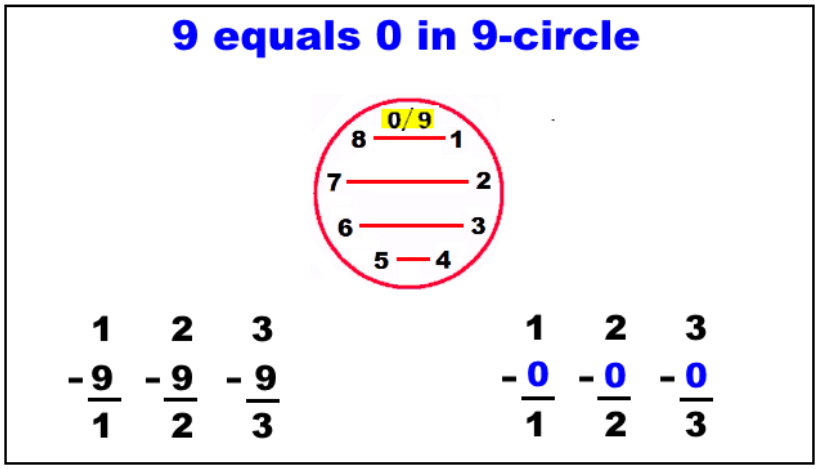
which is beginning to look more reasonable.

There are 3 rules which will take care of most of those remaining:

Rule 1

In 9-circle

0 = 9 and 9 = 0



Because 9 = 0 the sums on the left (1-9 = 1 etc.) are equivalent to the sums on the right (1 - 0 = 1 etc.)

There is no "9" in the list of 9-circle subtractions.
Just treat 9 as 0.

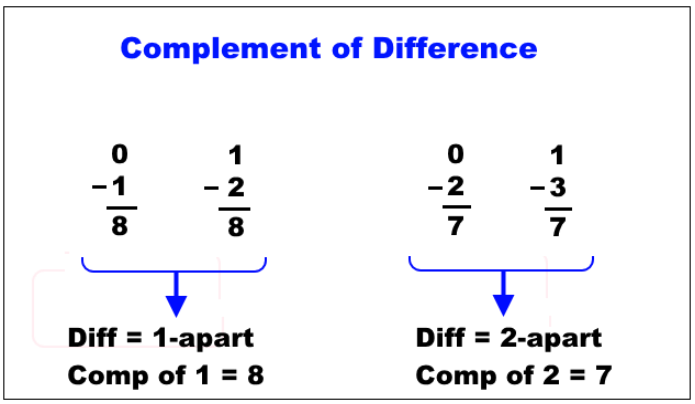
for the same reason
subtractions from zero are equivalent to
subtractions from nine.

Makes them easy doesn't it. We can remove this entire row from those that need to be memorized:

-	0	1	2	3	4	5	6	7	8
0		8	7	6	5	4	3	2	1
1			8	7	6	5	4	3	2
2				8	7	6	5	4	3
3					8	7	6	5	4
4						8	7	6	5
5							8	7	6
6								8	7
7									8

Rule 2

The "Complement of Difference" strategy
works in all circles - including 9-circle.



No new methods to learn.
Just apply the "Complement of the Difference" strategy
using 9-circle complements.

On the left (above) the numbers being subtracted are
1-apart
The complement of 1 in 9-circle is 8
8 is the answer.

Likewise on the right they are 2-apart
Complement and answer of 7.

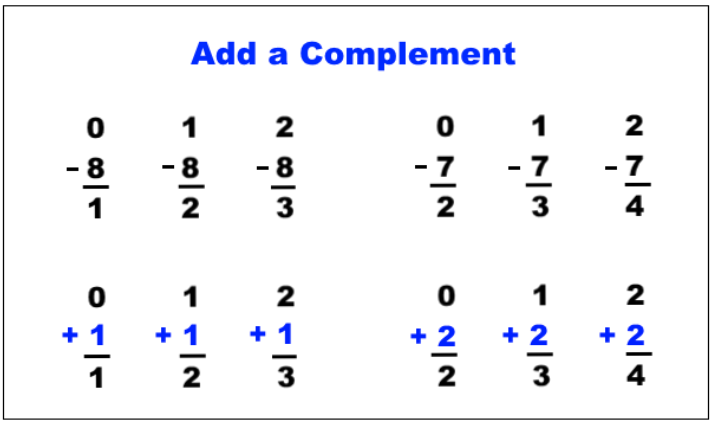
It's very easy to spot subtractions that are only 1 or 2 apart
The answers are just 8 or 7 in those cases.

We can remove all the subtractions
that are only 1 or 2 apart (coming to 7 and 8):

-	0	1	2	3	4	5	6	7	8
0									
1			8	7	6	5	4	3	2
2				8	7	6	5	4	3
3					8	7	6	5	4
4						8	7	6	5
5							8	7	6
6								8	7
7									8
8									

Rule 3

The "Add a Complement " strategy
works in all circles - including 9-circle.



It's ok to apply the "Add a Complement " strategy
using 9-circle complements.

The complement of 8 is 1 in 9-circle
So subtracting 8 becomes adding 1
0 - 8 = 0 + 1 = 1 etc.

Likewise subtracting 7 is the same as adding 2
0 - 7 = 0 + 2 = 2 etc.

All subtractions where we take away 7 or 8
can be eliminated:

-	0	1	2	3	4	5	6	7	8
0									
1					6	5	4	3	2
2						6	5	4	3
3							6	5	4
4								6	5
5									6
6									
7									
8									

What's Left?

We've eliminated 9's and 0's

subtractions which are only 1 or 2 apart

and subtractions which take away 7 or 8

because they can spotted and worked out on the fly.

That leaves this little batch of subtractions:

-	0	1	2	3	4	5	6	7	8
0									
1					6	5	4		
2						6	5		
3							6		
4									
5									
6									
7									
8									

Just 6 of them in all.

The strategies apply to them too.
It's just that they are not so easy to spot and distinguish.
Seeing 5 - 6 one can instantly spot "one apart"
and thus "complement 8" (answer).

But 2 - 5 is not quite so easy.
It's "three-apart"
"Complement 6" (answer).
It works - but not so instantaneously.

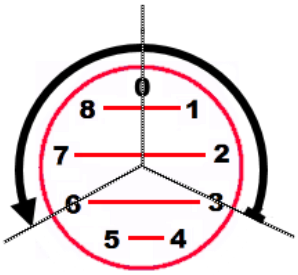
Draw Them

and Use Both Strategies

You can draw them in 9-circle to see how they work.

And also use both strategies on them.

For example here is 3 - 6 = 6:



3 - 6 = 6

3 - 6
= 3 + 3
= 6

3 - 6
Diff = 3
Comp = 6

Draw:

You can see starting from 3
counting backwards 6 steps
takes you back to 6 in 9-circle

Add a Complement:

The complement of 6 is 3
Instead of 3 - 6 we add 3 + 3 = 6

Complement of Difference:

The difference between 3 and 6 is 3
Complement of 3 is 6

You can do this with all 6 remaining results.

Patterns

There are some patterns that might help you recall these results:

-	4	5	6
1	6	5	4
2		6	5
3			6

1 - 4 = 6
1 - 5 = 5
1 - 6 = 4

2 - 5 = 6
2 - 6 = 5

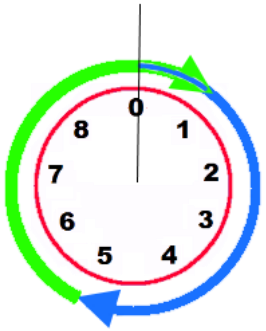
3 - 6 = 6

In 9-circle, begin at 0 and count round to 5 (see below in blue)

Then add on another 5 (in green):

5 + 5 = 10
5 + 5 = 1

10 = 9 + 1



You know that 5 + 5 = 10

and you can see that the 10 counts out to 1

around a 9-circle (see above).

So 5 + 5 = 1

That makes sense because

10 = 9 + 1

which is one loop back to 0 then on 1 more

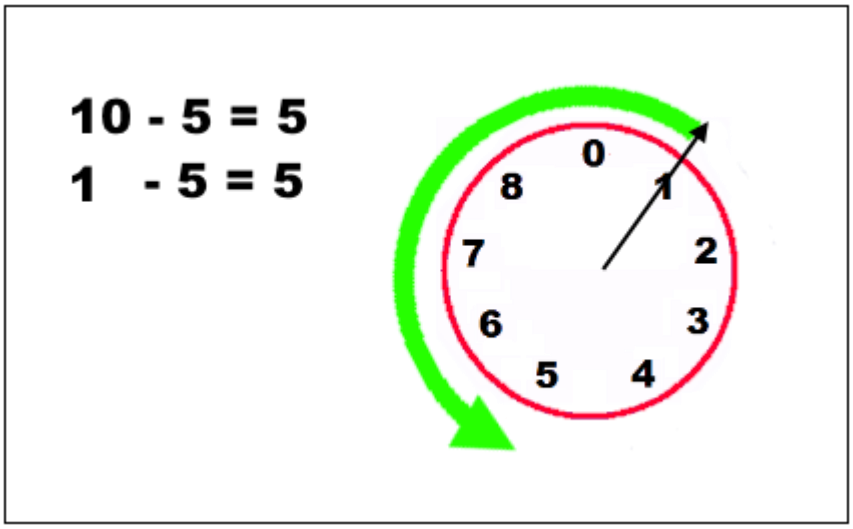
around the 9-circle.

So just like 9 = 0

we have 10 = 1 in a 9-circle.

Here is the result 1 - 5 = 5

which makes sense when we realize 10 comes to 1:



If you can remember that 1 acts like 10 in 9-circle

then:

1 - 4 = 6 *becomes* 10 - 4 = 6

1 - 5 = 5 *becomes* 10 - 5 = 5

1 - 6 = 4 *becomes* 10 - 6 = 4

and that just leaves:

2 - 6 = 5

2 - 5 = 6

3 - 6 = 6

It may help to notice that the answers are only 5 or 6

and that every sum

has 2 even numbers for every odd.

So 2 - 6 is already two even's

The answer must be 5 or 6

And it's odd.

It's 2 - 6 = 5.

Conclusion

I think you can see that it's not so difficult
to know ALL of the 9-circle subtractions.
(Work them out as you go or memorize a few).

You will also find that if you work the more difficult ones out a few times
that they quickly become memorized anyway.

These are the key to really rapid subtraction.

As it turns out they come in handy in other areas as well
(long addition and multiplication to be dealt with later)

So if you can take them on board
you will definitely be a lot better off.

But hey!
If this seems too much
Don't panic.
They represent icing on a cake.
You can still subtract pretty well without them.

That concludes this series.
Hopefully you will now be able to subtract
quickly, efficiently, easily
and in any direction whatsoever.

As time allows there will be more added to this website.
Thanks for looking into Circlemaths!